BASICS OF ACCEPTANCE SAMPLING

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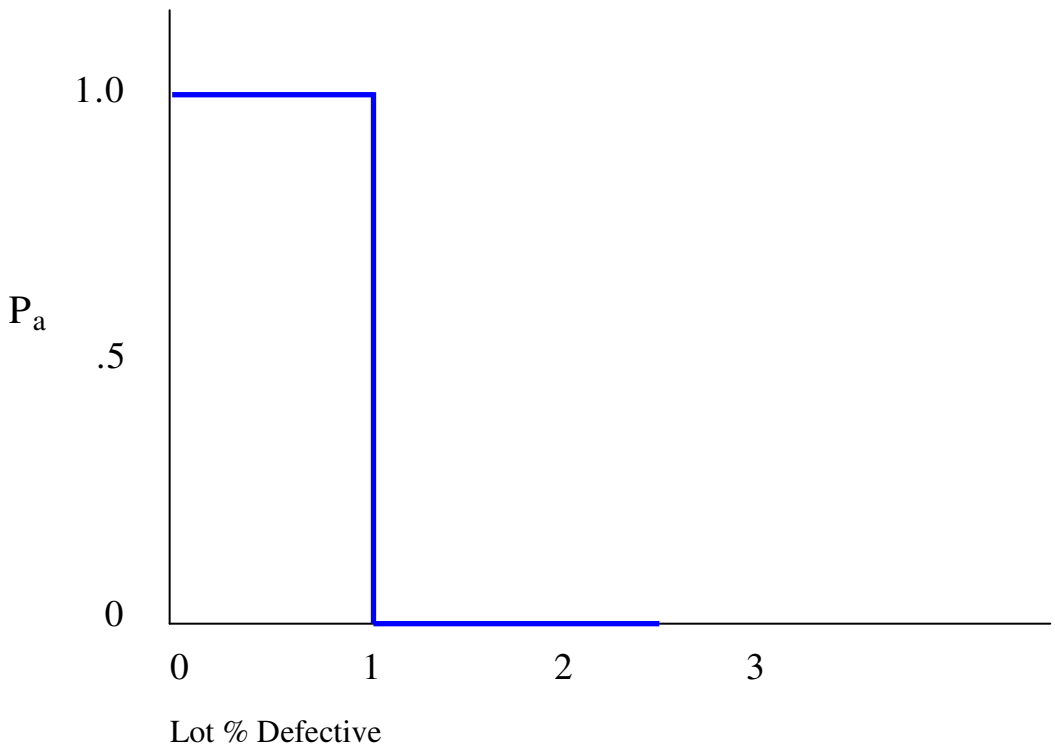
OPERATING CHARACTERISTIC (OC) CURVE

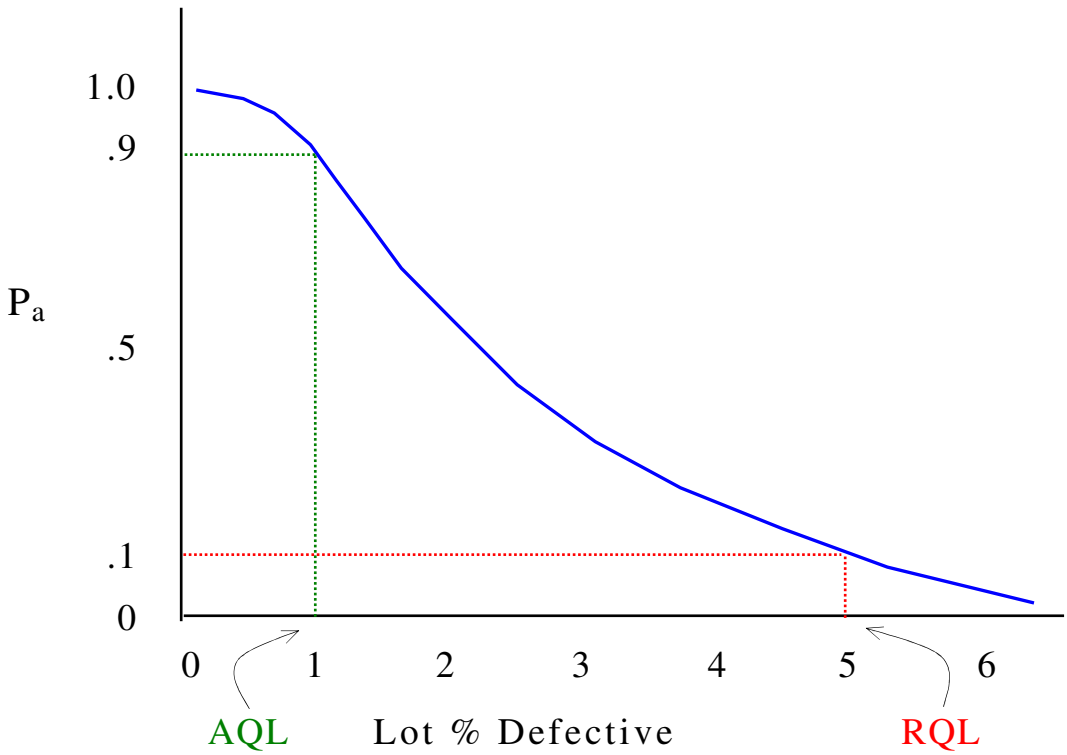
The OC curve quantifies the α and β risks of an attribute sampling plan. Below is an ideal OC curve (the bold line) for a situation in which we might want to accept all lots that are, say, ≤ 1% defective and reject all lots that are > 1% defective:



With this ideal (no risks) curve, all batches with < 1% defective incoming quality level would have a probability of acceptance (P_a) of 1.0. And, all lots with > 1% defective would have a P_a of 0. The P_a is the probability that the sampling plan will accept the lot; it is the long-run % of submitted lots that would be accepted when many lots of a stated quality level are submitted for inspection. It is the probability of accepting lots from a steady stream of product having a fraction defective p.

TYPICAL OC CURVE

Since there will always be some risks, a more typical looking OC curve looks more like the one below. It is based on the Poisson distribution* (with the defective rate < 10% and n is relatively large compared to N).



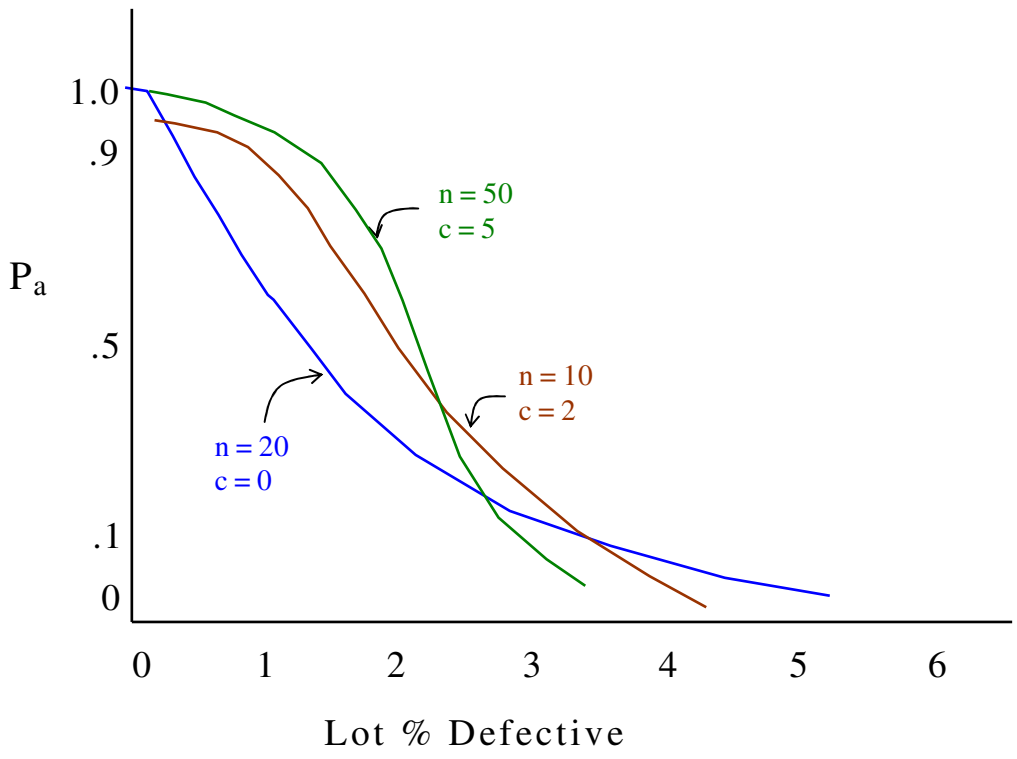
The AQL (Acceptance Quality Level), the maximum % defective that can be considered satisfactory as a process average for sampling inspection, here is 1%. Its corresponding P_a is about 89%. It should normally be at least that high.

The RQL (Rejectable Quality Level) is the % defective, here at 5%, that is associated with the established β risk (which is usually standardized at 10%). It is also known as the Lot Tolerance Percent Defective (LTPD).

*The hypergeometric and binomial distns are also used.

The alpha risk is the probability of rejecting relatively good lots (at AQL). The beta risk is the probability of accepting relatively bad lots (at LTPD/RQL). It is the probability of accepting product of some stated *un*desirable quality; it is the value of P_a at that stated quality level.

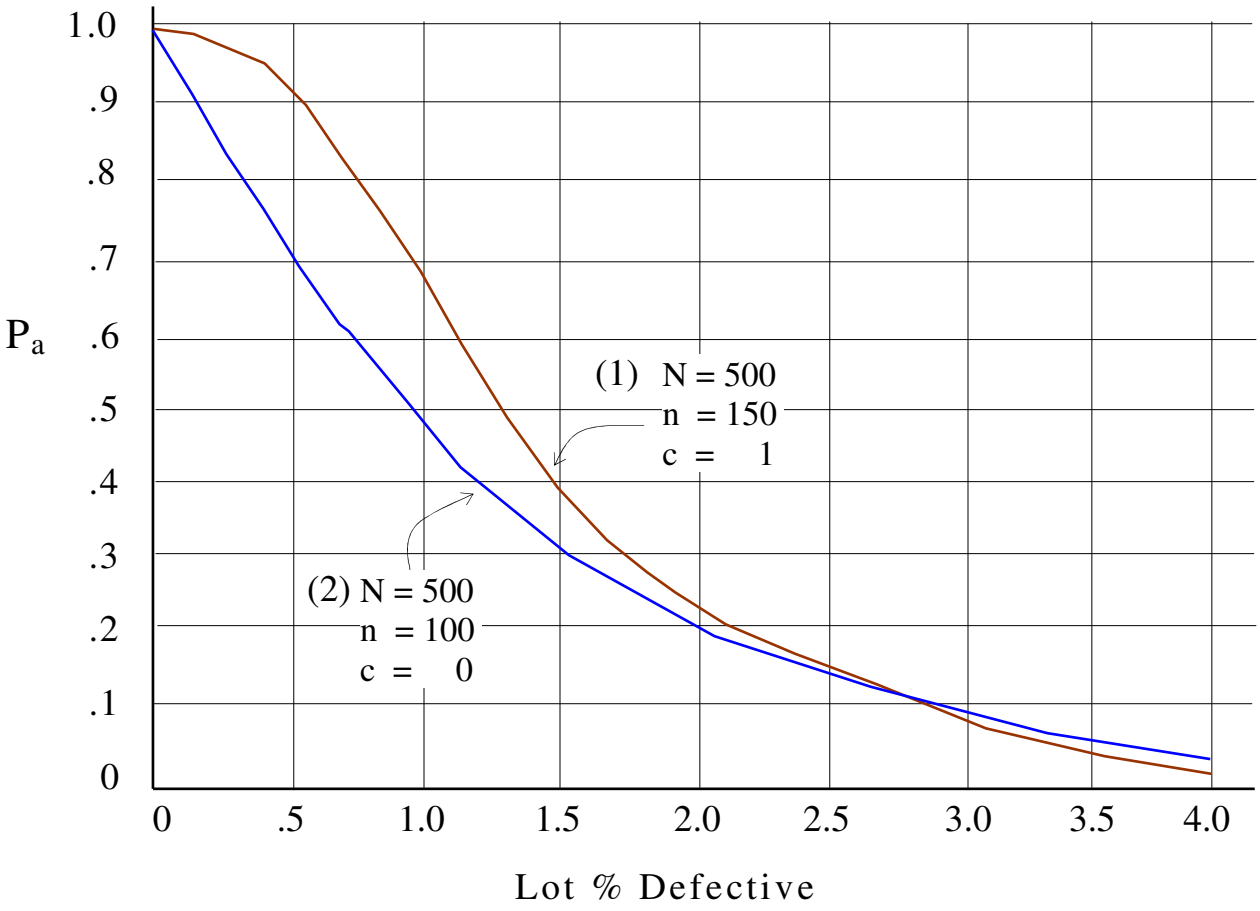
The OC curves are a means of quantifying alpha and beta risks for a given attribute sampling plan. The P_a value obtained assumes that the distribution of defectives among a lot is random – either the underlying process is in control, or the product was well mixed before being divided into lots. The samples must be selected randomly from the entire lot. The alpha risk is $1 - P_a$. The shape of the OC curves is affected by the sample size (n) and accept number (c) parameters. Increasing both the accept number and sample size will bring the curve closer to the ideal shape, with better discrimination.



If the lot size N changes, the above curves change very little. However, the curves will change quite a bit as sample size n changes. So, basing a sampling plan on a fixed percentage sample size will yield greatly different risks. For consistent risk levels, it is better to fix the sample size at n, even if the lot sizes N vary.

If n = 10 & c = 2, what is the alpha risk for a vendor running at p = .02? P_a is about .55, so alpha is about .45. What is the beta risk if the worst-case quality the customer will accept is 3%? (about 15%). To lower alpha & beta, you can increase n & c.

Is c = 0 the best plan for the producer and the consumer?



At the 2.8% lot defect rate, both plans give the producer equal protection: P_a = 11%, or $P_{rej.}$ = 89%. Which one gives better protection against rejecting relatively good lots, e.g., at the .5% lot defect rate, and why? For (1), α = about 8%; for (2), α = about 30%. (1) has a lower α error so less chance of rejecting good lots. With (2), you will reject any lot of 500 if there is even 1 defect in the sample, but it will lead to higher costs.

DISCRIMINATION IN ACCEPTANCE SAMPLING PLANS

Discrimination is the ability of a sampling plan to distinguish between relatively good levels of quality and relatively bad levels of quality, i.e., having

- A high P_a (e.g., 95%, 1-α) associated with a good level of quality P_1 (e.g., .5% or better)
- A low P_a (e.g., 10%, β) associated with a bad level of quality P_2 (e.g., 3% or worse)

The Operating Ratio is defined as

$$R = \frac{p_2}{p_1} = \frac{p_\beta}{p_{1-\alpha}}$$

Example

$$R = \frac{.03}{.005} = 6.0$$

DESIGNING YOUR OWN SINGLE ACCEPTANCE SAMPLING PLAN

Derive a plan that comes as close as possible to satisfying two points on the OC curve

The two points are (p_1, 1-α) and (p_2, β)

The derived plan will contain an n and a c value

Example

Desired α risk of .05 for a p_1 of .005, along with a desired risk of .05 for a p_2 of .03.

1. Determine R:

$$R = \frac{p_2}{p_1} = \frac{.030}{.005} = 6.0$$

2. Enter the Values of Operating Ratio Table with α and β and find the closest R to the calculated value in step 1.

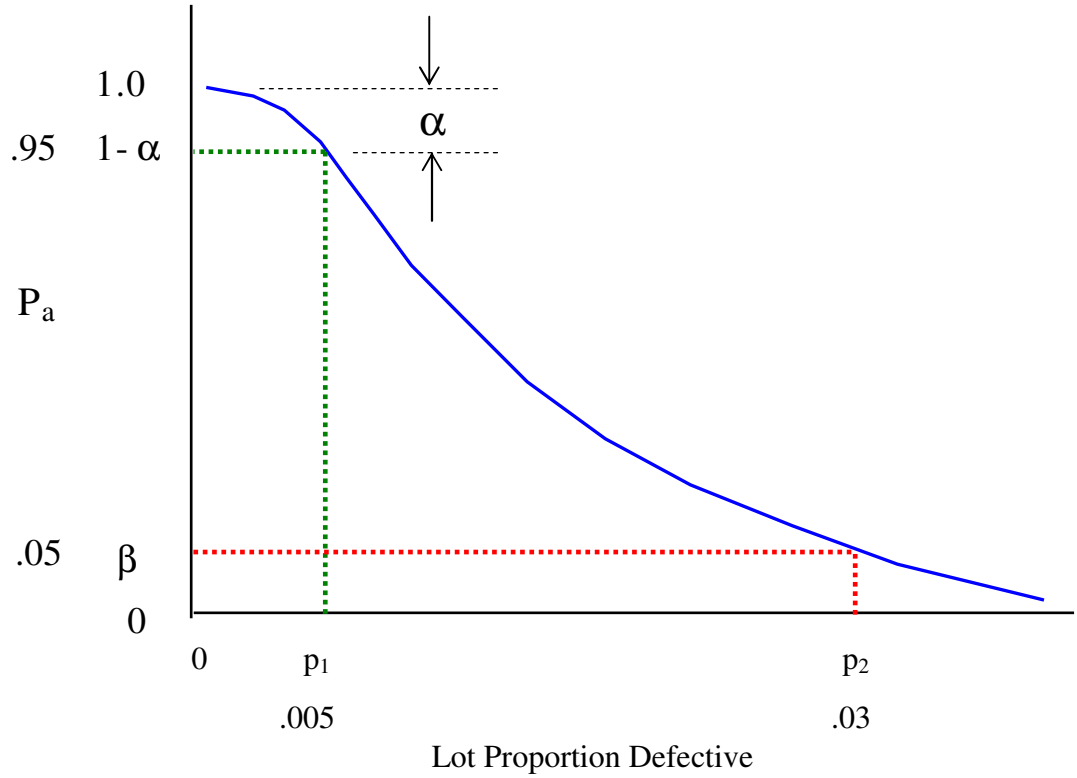
For α = .05 and β = .05, the closest table value is 5.67. This is acceptable since it is slightly more discriminating than 6.0. Note the c value of 3 in the far left column.

3. Obtain the np_1 value in the far right column. Then calculate n from:

$$n = \frac{n\,p_1}{p_1} = \frac{1.366}{.005} = 273.2 \rightarrow 274.$$

The acceptance sampling plan is n = 274, c = 3.

OC CURVE

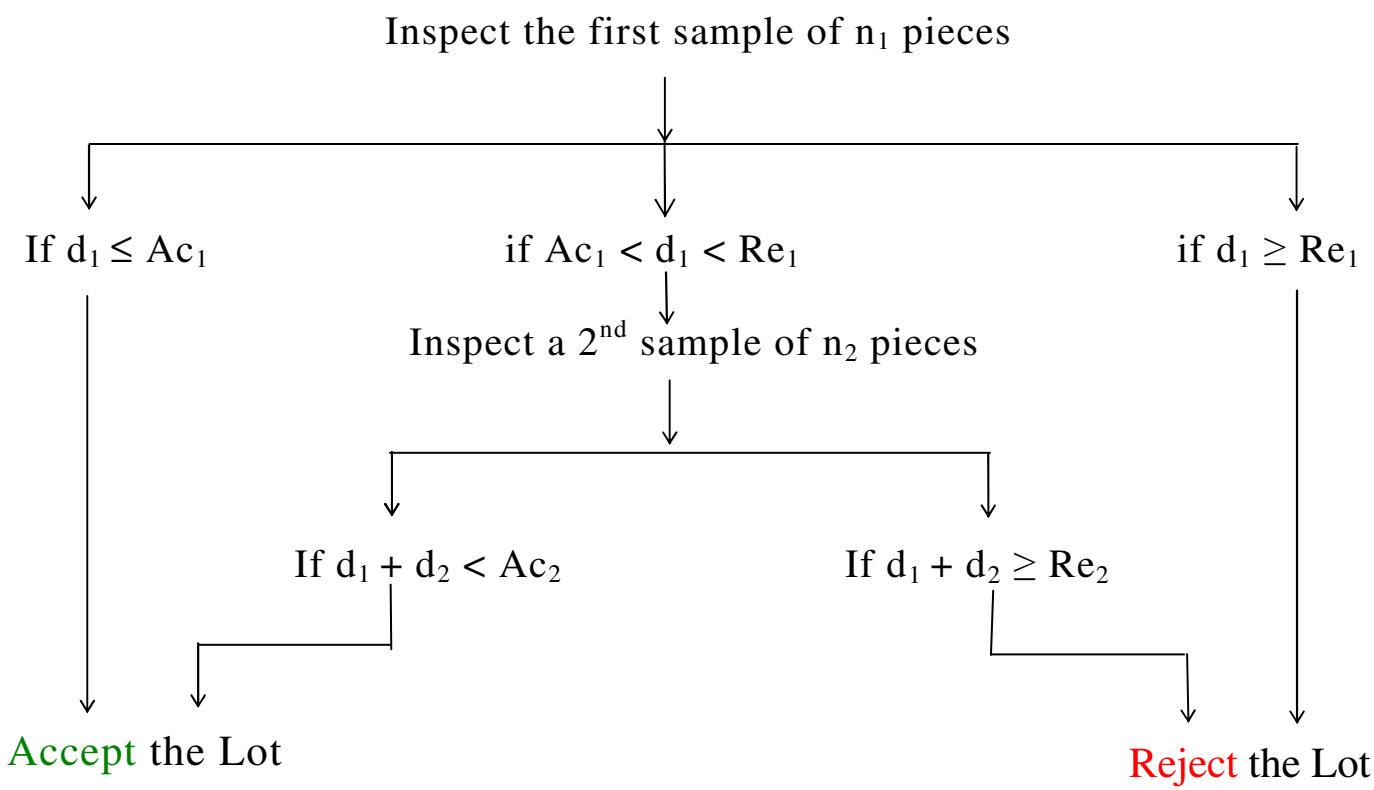


AQL BASED SAMPLING PLANS

- ANSI/ASQ Z1.4-1993 (MIL-STD 105E was withdrawn in February 1995) sampling plans are based on the use of *AQL* – the percent defective that is considered acceptable as a process average for the purposes of acceptance sampling.
- With these plans, it is *not* necessary to assume the provision of 100% screening (with replacement of all defective units) of all rejected lots.
- To enter any of these tables, you must first decide on the AQL to use (and determine the sample size code letter); from the table you will get the acceptance number (Ac) and the rejection number (Re) for the plan.
- There are also separate tables to provide the AOQL for different values of AQL and sample code letters
    - More serious defects should have a lower AQL as the acceptance criterion, and less serious defects can have higher AQLs – using a *Classification of Defects* system
    - *Tightened* inspection should be used whenever the quality history is unsatisfactory or when there are other good reasons for being suspicious about quality – keeping the beta risk down
    - *Reduced* inspection can be used when the quality history is shown to be good enough through *Normal* inspection

- These plans are generally chosen to protect the producer under normal conditions, i.e., not to reject submitted lots that are at the AQL or better when there are no reasons to be suspicious about quality

DOUBLE SAMPLING PLANS



- There is usually less sampling than for a single sampling plan
- The OC curve is better than the c = 0 curve for the single sampling plan with a smaller R (for better discrimination)

DESIGNING YOUR OWN DOUBLE ACCEPTANCE SAMPLING PLAN

Exercise

Desired α risk of .05 for a p_1 of .008, along with a desired risk of .10 for a p_2 of .06. Use Table 4-4.

1. Determine R:

$$R = \frac{p_2}{p_1} = \frac{.060}{.008} = 7.5$$

2. Enter Duncan's Double Sampling Tables and find the closest R to the calculated value in step 1.

The closest table value is 7.54 in Plan Number 2. This is very close to 7.5. Note the c_1 value of 1 and the c_2 value of 2.

3. For $Pa_{1-\alpha} = .95$, obtain the np_1 (pn_1 in the table) value. Then calculate n from:

$$n = \frac{n\,p_1}{p_1} = \frac{.52}{.008} = 65.$$

The acceptance sampling plan is $n_1 = 65$, $c_1 = 1$; $n_2 = 65$, $c_2 = 2$.

VARIABLES SAMPLING PLANS

ANSI/ASQ Z1.9 (MIL-STD-414 is withdrawn) sampling plans are based on the use of v*ariable* data (from an assumed normal distribution).

Actual measurements are made on the samples → the sample data is used to calculate a statistic, such as $\overline{X}$, R, or S and then the calculated statistic is compared to the critical value from a table.

Acceptance criteria must be applied *separately* to each quality characteristic (vs. overall lot accept vs. reject for attribute sampling), so it's more expensive than attribute sampling for larger lots, so it's generally best to use only on *key* characteristics, with attribute sampling on the rest.

Compared to attribute plans, these plans, for the same n, provide a greater quality protection in judging a single quality characteristic, or for the same amount of risk, a smaller n is OK.

The use of variable data can provide more information about the *extent* of nonconformity – How? (Hint: think frequency distribution and what you can do with this information).

These sampling procedures are based on the assumption that the quality characteristic is normally distributed (it is possible to use data transformation if it is not).

Procedure:

To use any of these plans, you must first decide on the:

- Inspection level (II is the default for Z1.9)
- Method to use – S or R, with variability (population sigma) known or unknown (see Decision Tree)
- AQL
- Lot size

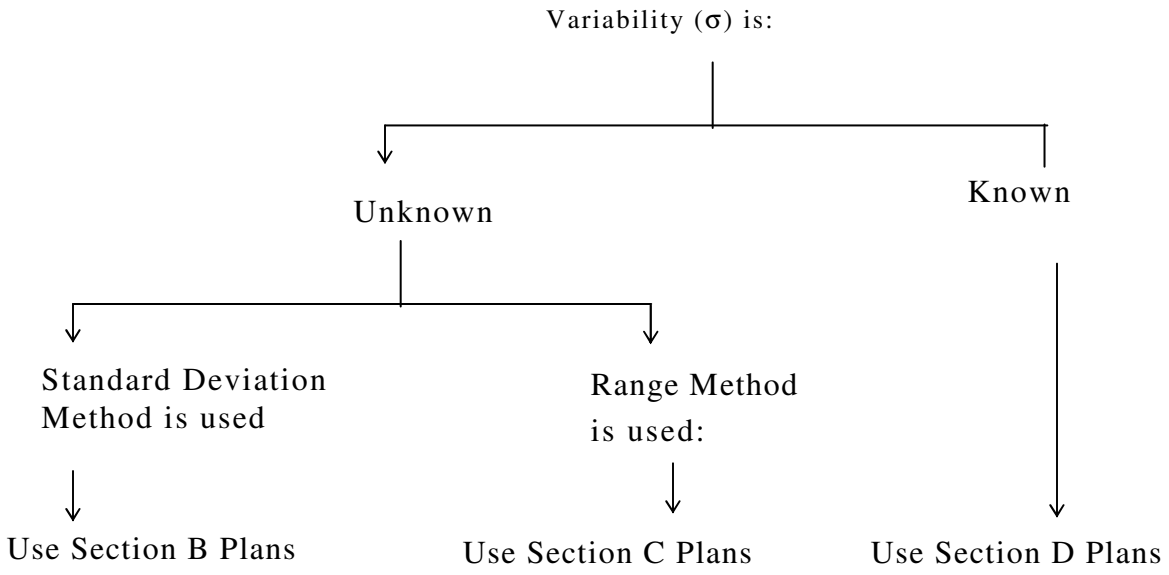
Determine the sample-size code letter from the table

Calculate the Q value

Enter the Master Table by sample size code letter and AQL, and look up the k value.

Compare the k and Q values – if the calculated Q value is ≥ than the critical k value from the table, accept the lot. If not, reject it.

Decision Tree for Selecting Type of Variables Sampling Plan



Determine the AQL to use in the Master Table: Since there are standard AQLs used in the Master Tables, you need to convert the AQL per table below:

For Specified AQL Values:	Use this AQL Value:
To .049	.10
.050-.069	.10
.070-.109	.10
.110-.164	.15
.165-.279	.25
.280-.439	.40
.440-.699	.65
.700-1.09	1.0
1.10-1.64	1.5
1.65-2.79	2.5
2.80-4.39	4.0
4.40-6.99	6.5
7.00-10.9	10.0

Determine the Sample-Size Code Letter to use in the Master Table:

ANSI/ASQ Z1.9 Table A.2

L O T SIZE	GENERAL I	INSPECTION II	LEVELS III
2-8	B	B	C
9-15	B	B	D
16-25	B	C	E
26-50	C	D	F
51-90	D	E	G
91-150	E	F	H
151-280	F	G	I
281-400	G	H	J
401-500	G	I	J
501-1200	H	J	K
1201-3200	I	K	L
3201-10000	J	L	M
10001-35000	K	M	N

Formulas for the Q Value:

For Section B – Standard Deviation Method (σ Unknown):

$$Q_U = \frac{USL - \bar{X}}{S} \quad \text{or } Q_L = \frac{\bar{X} - LSL}{S}$$

For Section C – Range Method (σ Unknown):

$$Q_U = \frac{USL - \bar{X}}{\bar{R}} \quad \text{or } Q_L = \frac{\bar{X} - LSL}{\bar{R}}$$

For Section D – Standard Deviation Method (σ Known):

$$Q_U = \frac{USL - \bar{X}}{\sigma} \quad \text{or } Q_L = \frac{\bar{X} - LSL}{\sigma}$$

Note: For 2-sided spec limits, an AQL can be assigned to both limits combined, or to each end of the spec limit separately.

Table B-1 Standard Deviation Method Master Table B-1 for Normal and Tightened Inspection for Plans Based on Variability Unknown (Single Specification Limit)

Obtain the k Value from the Master Table:

		Acceptance Quality Levels (Normal Inspection) ↓									
Code Letter	Sample Size	.10 k	.15 k	.25 k	.40 k	.65 k	1.00 k	1.50 k	2.50 k	4.00 k	6.50 k
B	3	⇩	⇩	⇩	⇩	⇩	⇩	⇩	1.12	.958	.765
C	4	⇩	⇩	⇩	⇩	⇩	1.46	1.34	1.17	1.01	.815
D	5	⇩	⇩	⇩	1.77	1.65	1.52	1.40	1.24	1.07	.874
E	7	2.22	2.13	2.00	1.88	1.75	1.62	1.50	1.33	1.15	.955
F	10	2.34	2.24	2.11	1.98	1.84	1.72	1.59	1.41	1.23	1.03
G	15	2.42	2.32	2.19	2.06	1.92	1.79	1.65	1.47	1.30	1.09
H	20	2.47	2.37	2.23	2.10	1.96	1.83	1.69	1.51	1.33	1.12
I	25	2.50	2.40	2.26	2.13	1.98	1.85	1.72	1.53	1.35	1.14
J	35	2.55	2.45	2.31	2.18	2.03	1.89	1.76	1.55	1.39	1.19
K	50	2.61	2.50	2.36	2.22	2.08	1.94	1.80	1.57	1.42	1.21
L	75	2.66	2.55	2.41	2.27	2.12	1.98	1.84	1.58	1.96	1.25
M	100	2.69	2.58	2.43	2.29	2.14	2.00	1.86	1.61	1.48	1.26
N	150	2.73	2.62	2.47	2.33	2.18	2.03	1.89	1.65	1.51	1.29
P	200	2.73	2.62	2.47	2.33	2.18	2.04	1.89	1.70	1.51	1.29
		.15	.25	.40	.65	1.00	1.50	2.50	4.00	6.50	--
		AQL (Tightened Inspection) ↑									

DESIGNING YOUR OWN VARIABLE SAMPLING PLAN

For a known population sigma:

$$n = \left[\frac{(Z_\alpha + Z_\beta)}{(Z_1 - Z_2)}\right]^2$$

For an unknown population sigma:

$$n = (1 + k^2/2)\left[\frac{(Z_\alpha + Z_\beta)}{(Z_1 - Z_2)}\right]^2 \quad \text{Where} \quad k = \frac{Z_2 Z_\alpha + Z_1 Z_\beta}{Z_\alpha + Z_\beta}$$

Exercise

What would be the variables sampling plan (sigma unknown) for the following conditions? High P_a ($\alpha = .05$) for a fraction non-conforming (P_1) of .005, with a low P_a ($\beta = .05$) for a fraction conforming (P_2) of .03.

Solution

$Z_\alpha = Z_{.05} = 1.645$ $\quad Z_\beta = Z_{.05} = 1.645$

$Z_1 = Z_{.005} = 2.576$ $\quad Z_2 = Z_{.03} = 1.881$

$$k = \frac{1.881(1.645) + 2.576(1.645)}{1.645 + 1.645} = 2.23$$

$$n = \left(1 + 2.23^2/2\right)\left[\frac{1.645 + 1.645}{2.576 - 1.881}\right]^2 = 78.128 \rightarrow 79$$

Note: with the same operating characteristics, an attribute sampling plan would require n = 274.

Example (Method: Population Sigma Known)

A lot of 1500 bobbins is submitted for inspection. Inspection level II, normal inspection, with AQL = .65%, is to be used. The specified minimum yield value for the tensile strength is 25.0 lbs. The variability σ is known to be 2.4 lbs.

The sample size code letter from Table A.2 is K. In Table D-2 (p. 86 in ANSI/ASQ Z1.9-2003), for reduced inspection, the required sample size is 7 and the k value is 1.80. The 7 sample specimen's tensile strengths are 25.7, 26.4, 26.1, 27.2, 25.8, 28.3, and 27.4.

$$Q_L = \frac{\bar{X} - LSL}{\sigma} = \frac{26.7 - 25.0}{2.4} = .63$$

Since $Q_L < k$, the lot does not meet the acceptability criterion and should be rejected.

What are the alpha and beta risks for sampling letter K (p. 23 in ANSI/ASQ Z1.9-2003), for an AQL of .65, for various incoming quality levels (P)?

	P	P_a	α	β
Relatively Good Quality	.25	99.5	.5	
	.50	96	4	
Marginal	.75	90	10	90
	1.50	62	38	62
Poor	2.00	46		46
	3.00	22		22
	4.00	10		10
	5.00	4.5		4.5

* All values are in Percentages

Why is the lot rejected even though none of the samples were out of spec? (Assuming this is a representative sample, a larger, +/- 3σ distribution would provide some product out of specification; in this case, in the left tail of the distribution.)

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